

$$Q(x) > 2x$$

$$\frac{L}{g} = 2.2$$

$$\frac{dV}{dx} + P_1^E(x) V = Q_1(x)$$

$$-L \frac{dy}{y^2} = \frac{dx}{x}$$

$$\text{I.F.} = e^{\int -2x \, dx} = e^{-x^2}$$

$$\text{Sol: } \int -2x e^{-x^2} dx = \int -2x e^{-x^2} dx + C$$

$$x^2 = t$$

$$2x \, dx = dt$$

$$= \int e^{-t} dt + C$$

$$y = e^{-x^2} \Rightarrow e^{-x^2} + C$$

$$\frac{1}{y} = 1 + ce^{x^2} \quad \underline{\underline{\text{Ans}}}$$

Derick Recall from Lecture 24

$$\left| \frac{dy}{dx} \right| + |y| > 0 \quad \text{wobei} \quad y(0) = 1$$

has $y > 0$ as a sol. of D-E. but not of E-V. P.

Q:1) Under what condⁿ. an FVP has atleast one solⁿ?

Q-2) " " " " " has Unique solⁿ.

when it is difficult to solve a DE, we may be interested to get an idea if sol. exists at all or not.

Picard chosen: Let DE be $\frac{dy}{dx} = f(x, y)$ $y(x_0) = y_0$

en $R : |x - x_0| \leq a, |y - y_0| \leq b$

if 1) $f(x, y)$ is conti. in $\mathbb{R}$

2) $f(x,y)$ is bdd in R i.e. $|f(x,y)| \leq M$

then $\exists$ a sol. $y(x)$ in interval $|x-x_0| \leq h$ where
 $\uparrow$
 there exists $\boxed{h = \min(a, b/M)}$

Q. Find the interval in which $y' = 2x^2 + 3y^2$ has a sol.
 $y(0) = 1$
 in $R: |x-0| \leq 1, |y-1| \leq 1 \rightarrow \begin{matrix} -1 \leq y-1 \leq 1 \\ 0 \leq y \leq 2 \end{matrix}$

Ans $f(x,y) = 2x^2 + 3y^2$

i) $f(x,y)$ is a polynomial & hence conti.

ii) $|f(x,y)| = |2x^2 + 3y^2| \leq |2x^2| + |3y^2|$

$$\leq 2|x|^2 + 3|y|^2$$

$$|x+y| \leq |x| + |y|$$

$$\leq 2 \cdot 1^2 + 3 \cdot 2^2 = 2 + 12 = 14 = M$$

$\therefore \exists$ a sol. in $|x-0| \leq h$ where $h = \min(1, 1/14) = 1/14$

$\boxed{|x| \leq 1/14}$ Ans.

Lipschitz condition: A f^n . $f(x,y)$ is said to satisfy
 Lipschitz condⁿ in R if $\exists K > 0$ s.t.

$$|f(x,y_1) - f(x,y_2)| \leq K |y_1 - y_2|$$

Uniqueness thm: Given $y' = f(x,y)$ $y(x_0) = y_0$ in R
 $|x-x_0| \leq a, |y-y_0| \leq b$

if 1) f is conti. in R

2) $|f(x,y)| \leq M$ i.e. f is bdd in R

3) f satisfies L.C.

then $\exists$ a unique sol. in $|x-x_0| \leq h$ where $h = \min(a, b/M)$

Q.1. $f(x,y) = 2x^2 + 3y^2$

$$|f(x,y_1) - f(x,y_2)| = |2x^2 + 3y_1^2 - 2x^2 - 3y_2^2|$$

Q.1. $f(x, y) = 2x^2 + 3y^2$

$$|f(x, y_1) - f(x, y_2)| = |2x^2 + 3y_1^2 - 2x^2 - 3y_2^2|$$

$$= |3y_1^2 - 3y_2^2|$$

$$= 3 |y_1^2 - y_2^2|$$

$$= 3 |(y_1 - y_2)(y_1 + y_2)|$$

$$= 3 |y_1 - y_2| |y_1 + y_2|$$

$$\leq 3 (\underbrace{|y_1| + |y_2|}) |y_1 - y_2|$$

$$\because 0 \leq y \leq 2$$

$$\leq 3 (2 + 2) |y_1 - y_2|$$

$$\leq 12 |y_1 - y_2|$$

$$K = 12 > 0$$

Q.2) $y' = 6x^2 y^2$ $y(0) > 0$ $R: |x| \leq 2$ $|y| \leq 3$

1) $f(x, y) = 6x^2 y^2$ is polynomial hence contin

2) $|f(x, y)| = |6x^2 y^2| = 6 |x|^2 |y|^2 \leq 6 \cdot 2^2 \cdot 3^2 = 216$
 $= M$

3) $|f(x, y_1) - f(x, y_2)| = |6x^2 y_1^2 - 6x^2 y_2^2|$

$$= 6 |x|^2 |y_1^2 - y_2^2|$$

$$= 6 |x|^2 |y_1 + y_2| |y_1 - y_2|$$

$$\leq 6 |x|^2 (|y_1| + |y_2|) |y_1 - y_2|$$

$$= 6 \cdot 2^2 (3 + 3)$$

$$= 144 |y_1 - y_2|$$

$$K = 144 > 0$$

$\therefore$ a unique sol. exist $|x - x_0| \leq h$ i.e. $|x| \leq h$

$$h = \min(a, b/M) = \left(2, \frac{3}{216}\right) = \frac{3}{216}$$

$$|x| \leq \frac{3}{2/6}$$

Ans.